

W⁺AM 2024 Yearbook

Symmetry and Arithmetic



Institute for Advanced Study, Princeton

May 19-24, 2024

Funding for the program is provided by the National Science Foundation, Institute for Advanced Study, the Robert S. Hillas Fund, Princeton University and Lisa Simonyi.

This is a compilation of activities and resources contributed by participants during the 2024 Women⁺ and Mathematics Program. We hope this can serve as a mathematical and professional reference guide for women mathematicians around the country.

Mathematical Talks

Terng Lectures:

Charlotte Chan, University of Michigan, "Deligne-Lusztig theory: examples and applications"



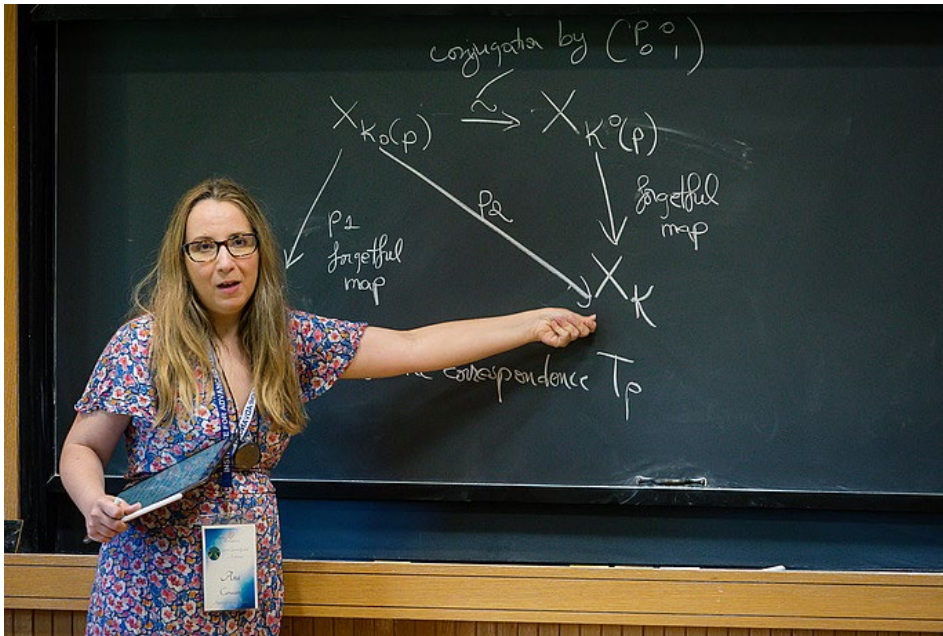
Geometry and representation theory are intertwined in deep and foundational ways. One of the most important instances of this relationship was uncovered in the 1970s by Deligne and Lusztig: the representation theory of matrix groups over finite fields is encoded in the geometry of a natural "partition" of flag varieties. Recent developments have revealed rich connections between Deligne-Lusztig varieties and geometry studied in number-theoretic contexts. In this lecture series, we give an example-based tour of these ideas, focusing on how to extract concrete information from theory.

Terng problem sessions were run by Si Ying Lee, Harvard University.



Uhlenbeck Lectures:

Ana Caraiani, Imperial College London, "A glimpse into the Langlands program"



Abstract: The goal of this lecture series is to give you a glimpse into the Langlands program, a central topic at the intersection of algebraic number theory, algebraic geometry and representation theory. In the first lecture, we will look at a celebrated instance of the Langlands correspondence, namely the modularity of elliptic curves. I will try to give you a sense of the different meanings of modularity and of the multitude of ingredients that go into establishing such a result. In the following lectures, I will focus on the more geometric ingredients, first in the special case of the modular curve and then for higher-dimensional Shimura varieties.

Uhlenbeck Problem Sessions were run by Alice Pozzi, University of Bristol.



References for both courses:

For the Uhlenbeck Lecture:

1. Serre's chapter on modular forms in "A course in arithmetic".
2. Fred Diamond and Jerry Shurman "A first course in modular forms"
3. The video of Frank Calegari's plenary ICM address in 2022:
<https://www.youtube.com/watch?v=EDsK-8SBx-g>
4. The article "Langlands reciprocity: L-functions, automorphic forms, and Diophantine equations" by Matt Emerton that was published in a collection called "The genesis of the Langlands program". (This is more for a historical perspective and one does not need to follow everything in the article, but it can be used as a guide for how different objects are

supposed to be related.) The article is available

here: <https://math.uchicago.edu/~emerton/pdffiles/reciprocity.pdf>

5. "Fermat's last theorem" by Diamond, Darmon and Taylor, available

here: <https://www.math.mcgill.ca/darmon/pub/Articles/Expository/05.DDT/paper.pdf>

For more advanced references:

1. Scholze (ICM 2018), available here: <https://people.mpim-bonn.mpg.de/scholze/Rio.pdf>

2. Thorne (ECM 2022), available here: <https://content.ems.press/assets/public/full-texts/books/262/chapters/online-pdf/978-3-98547-551-3-chapter-5175.pdf>

3. "Recent progress on Langlands reciprocity for GL_n : Shimura varieties and beyond", that I wrote jointly with Sug Woo Shin, available here:

<https://www.ma.imperial.ac.uk/~acaraian/papers/IHES-Caraiani-Shin.pdf>

4. "Perfectoid spaces: lectures from the 2017 Arizona Winter School".

For the Terng Lecture:

Books:

1. Linear Representations of Finite Groups (Serre)

2. Representations of $SL_2(F_q)$ (Bonnafe)

3a. Representations of Finite Chevalley Groups (Srinivasan)

3b. Representations of Finite Groups of Lie Type (Digne, Michel)

3c. The Character Theory of Finite Groups of Lie Type (Geck, Malle)

4. Characters of Reductive Groups over a Finite Field (Lusztig)

5. The Local Langlands Conjecture for $GL(2)$ (Bushnell, Henniart)

Papers:

1. The characters of the finite symplectic group $Sp(4, q)$ (Srinivasan)

2. Representations of reductive groups over finite fields (Deligne, Lusztig)

3. Représentations linéaires des groupes finis "algébriques", d'après Deligne-Lusztig (Serre)

4. Character sheaves, I (Lusztig)

5. Character sheaves (Mars, Springer)

6. Reductive groups over local fields (Tits)

7a. Unrefined minimal K -types for p -adic groups (Moy, Prasad)

7b. Jacquet functors and unrefined minimal K -types (Moy, Prasad)

8. On non-abelian Lubin-Tate theory via vanishing cycles (Yoshida)

Colloquium

Jessica Fintzen, University of Bonn, "An Introduction to Representations of p -adic Groups"



Research Seminar

Mathilde Gerbelli-Gauthier, McGill University

Title: Growth of Cohomology of Picard Modular Surfaces: An Illustrated Example of Langlands Functoriality

Abstract: How fast do Betti numbers grow in a congruence tower of covering spaces? I'll discuss this question in the special case of Picard modular surfaces, which are 4-dimensional real manifolds. There, the question is most interesting in degree 1, for which there is expected to be very little cohomology. I'll explain how the problem is related to automorphic forms, and specifically how the dearth of cohomology classes is a consequence of Langlands functoriality.

Kalyani Kansal, IAS

Title: Emerton-Gee Stack for GL_2 and Categorical p -adic Langlands

Abstract: The Emerton-Gee stack is a stack of étale ϕ modules, and can be viewed as a stack of p -adic representations for the Galois group of a finite extension K of \mathbb{Q}_p . In this talk, we will introduce the stack and talk about its role in the conjecture by Emerton, Gee and Hellmann on categorical p -adic Langlands. We will focus attention on the reduced part of the stack for GL_2 , which can be viewed as parametrizing two-dimensional representations with mod p coefficients, and discuss various questions and results motivated by this conjectural categorical correspondence.



Si Ying Lee, Harvard University

Title: Integral Hecke Correspondences

Abstract: Shimura varieties are an important geometric object in the Langlands program, because the Hecke (adelic group) action allows us to view various cohomology groups as Hecke modules. Moreover, the cohomology admits an integral structure when the Shimura variety has an integral model, and we hence want to extend this Hecke action integrally. I will talk about constructing integral models for Hecke correspondences in both the global and local (Rapoport-Zink spaces) setting, to get quasi-smooth derived (formal) schemes. This is joint work in progress with Keerthi Madapusi.

Alice Pozzi, University of Bristol

Title: Modular Generating Series for Real Quadratic Heegner Objects

Abstract: The theory of elliptic curves with complex multiplication has yielded some striking arithmetic applications, ranging from (cases of) Hilbert's Twelfth Problem to the Birch and Swinnerton-Dyer Conjecture. These applications rely on the construction of certain "Heegner objects", arising from imaginary quadratic points on the complex upper half plane; the most famous examples of these are Heegner points.

In recent years, conjectural analogues of these Heegner objects for real quadratic fields have been constructed via p -adic methods. In this talk, I will discuss how Heegner objects for real quadratic fields can be used to obtain modular generating series, that is, formal q -series that are q -expansions of classical modular forms. This is joint work with Judith Ludwig, Isabella Negrini, Sandra Rozensztajn and Hanneke Wiersema.

WAM Ambassadors

More information on how to apply for the WAM Ambassadorships please go here:

<https://www.ias.edu/math/wam/apply/ambassador-program>

